

## “Additional Loss due to SWR”

Definitions:

$Z_{load}$	complex load impedance
$Z_{in}$	complex line input impedance
$Z_0$	complex line characteristic impedance
$l$	line length
$\gamma$	complex line attenuation constant
$\rho_{load}$	complex reflection coefficient looking into the load
$\rho_{in}$	complex reflection coefficient looking into the line
$I_{in}$	complex current flowing into the line
$I_{load}$	complex current flowing into the load

From Kumar “Microwave techniques - Transmission Lines” page 86; dividing equation (5.69) by equation (5.68) we get:

$$\frac{I_{load}}{I_{in}} = \left[ \frac{1 - \rho_{load}}{1 - \rho_{in}} \right] \cdot e^{(-\gamma l)}$$

The term  $e^{(-\gamma l)}$  represents the current attenuation into a matched load, so  $\left[ \frac{1 - \rho_{load}}{1 - \rho_{in}} \right]$  represents the additional multiplying factor attributable to the mismatched condition.

This additional current multiplying factor - call it  $X$  - can be expanded and simplified as follows:

$$\begin{aligned} X &= \frac{1 - \rho_{load}}{1 - \rho_{in}} \\ X &= \frac{1 - \frac{Z_{load} - Z_0}{Z_{load} + Z_0}}{1 - \frac{Z_{in} - Z_0}{Z_{in} + Z_0}} \\ X &= \frac{\frac{2 \cdot Z_0}{Z_{load} + Z_0}}{\frac{2 \cdot Z_0}{Z_{in} + Z_0}} \\ X &= \frac{Z_{in} + Z_0}{Z_{load} + Z_0} \end{aligned}$$

To find the additional power multiplication factor, we must square  $X$  and also take account of the fact that  $I_{in}$  and  $I_{load}$  are flowing into different values of resistance. We then get the additional power multiplying factor as:

$$\frac{R_{load}}{R_{in}} \cdot \left| \frac{Z_{in} + Z_0}{Z_{load} + Z_0} \right|^2 \quad \text{or, expressed as additional loss (dB):} \quad -10 \cdot \text{Log} \left( \frac{R_{load}}{R_{in}} \cdot \left| \frac{Z_{in} + Z_0}{Z_{load} + Z_0} \right|^2 \right)$$

where  $R_{in} = \Re(Z_{in})$  and  $R_{load} = \Re(Z_{load})$